

# Decay coupling constant sum rules for tetraquarks $T[(\bar{Q}q)(Q\bar{q})]$ with broken SU(3) symmetry

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## Abstract

For tetraquarks of the form  $T[(\bar{Q}q)(Q\bar{q})]$  we give sum rules for their decay coupling constants, taking into account the SU(3) symmetry breaking interactions to first order.

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## I. INTRODUCTION

Two hidden-bottom charged meson resonances,  $Z_b(10610)$  and  $Z_b(10650)$  with  $J^P = 1^+$ , were observed recently by the Belle Collaboration [1]. Since these decay into  $\pi^\pm \Upsilon(nS)$  ( $n = 1, 2, 3$ ) and  $\pi^\pm h_b(mP)$  ( $m = 1, 2$ ) they have been interpreted [2–4] as tetraquarks of the form  $T(Q\bar{Q}q\bar{q})$ . A natural decay mode for such a tetraquark would be

$$T(Q\bar{Q}q\bar{q}) \rightarrow H(Q\bar{Q}) + M(q\bar{q}), \quad (1)$$

where  $H(Q\bar{Q})$  is a heavy meson with  $Q = b$  or  $c$  and  $M(q\bar{q})$  could be the pseudoscalar or vector meson SU(3) flavour octet. For such tetraquarks we recently [5] gave sum rules for tetraquark decay coupling constants taking into account the SU(3) breaking interactions to first order.

In this paper we consider tetraquarks of the form  $T[(\bar{Q}q)(Q\bar{q})]$  where the heavy quark  $Q = b$  or  $c$  and the light quarks  $q = u, d$ , or  $s$ . For a given  $Q$  there will be nine such tetraquarks. The natural decay mode will be into two mesons  $(\bar{Q}q)$  and  $(Q\bar{q})$  subject to angular momentum and parity selection rules. For example, a  $J^P = 0^+$  tetraquark would decay into two pseudoscalar mesons.

We consider the decays

$$T[(\bar{Q}q)(Q\bar{q})] \rightarrow M(\bar{Q}q) + M(Q\bar{q}), \quad (2)$$

in Sec. III below. We give sum rules for the nine decay couplings constants taking into account the SU(3) breaking interactions to first order.

An important difference between the tetraquarks  $T(Q\bar{Q}q\bar{q})$  considered earlier and the tetraquarks  $T[(\bar{Q}q)(Q\bar{q})]$  considered here is that the latter can have OZI [6–8] suppressed decays through annihilation of  $\bar{Q}^\alpha Q_\beta$  through gluons into lighter quarks  $\bar{q}^\alpha q_\beta$  (greek letters represent the color indices). These modes are briefly discussed in the concluding remarks.

## II. NOTATION

We denote the tetraquark made of a heavy meson and its antiparticle as

$$T_k^i = T[(\bar{Q}q_k)(Q\bar{q}^i)], \quad (3)$$

with  $i, k = 1, 2, 3$  (latin letters representing flavour indices). The heavy quark  $Q = b$  or  $c$  is SU(3) flavour singlet. The light quarks  $q_k$  ( $k = 1, 2, 3$ ) transform as SU(3) **3** while the antiquarks  $\bar{q}^i$  ( $i = 1, 2, 3$ ) transform as  $\bar{\mathbf{3}}$ . In color space,  $M_k \equiv (\bar{Q}q_k)$  and  $\bar{M}^i \equiv (Q\bar{q}^i)$  form color singlets and transform as **3** and  $\bar{\mathbf{3}}$  of SU(3) flavour, respectively.

The natural decay mode for the tetraquark would be

$$T_k^i \rightarrow M_k + \bar{M}^i. \quad (4)$$

The tetraquark states can be represented as a  $3 \times 3$  matrix  $T$  with  $T_k^i$  as matrix elements, so that,

$$T = \begin{pmatrix} T_1^1 & T_1^2 & T_1^3 \\ T_2^1 & T_2^2 & T_2^3 \\ T_3^1 & T_3^2 & T_3^3 \end{pmatrix}. \quad (5)$$

In  $SU(3)$  flavour space this transforms as  $\mathbf{1} \oplus \mathbf{8}$ . The mesons  $M_k$  and  $\bar{M}^i$  transform as  $SU(3)$   $\mathbf{3}$  and  $\bar{\mathbf{3}}$ , respectively. The  $3 \times 3$  matrix  $\mathcal{M}$  representing the final states is

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_1^1 & \mathcal{M}_1^2 & \mathcal{M}_1^3 \\ \mathcal{M}_2^1 & \mathcal{M}_2^2 & \mathcal{M}_2^3 \\ \mathcal{M}_3^1 & \mathcal{M}_3^2 & \mathcal{M}_3^3 \end{pmatrix}, \quad (6)$$

where  $\mathcal{M}_k^i$  represents  $M_k + \bar{M}^i$  in the final state. In flavour space it also transforms as  $\mathbf{1} \oplus \mathbf{8}$ , this is clear since  $(\bar{Q}q) \otimes (Q\bar{q})$  transform as  $\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \mathbf{8}$ .

### III. SUM RULES

In unbroken  $SU(3)$  there will be a single decay coupling constant  $G_0$ ; with  $\lambda_8$  breaking the tetraquark octet can form two octets, symmetric (coupling constant  $G_D$ ) and antisymmetric (coupling constant  $G_F$ ). In matrix form we can write the decay coupling constant as

$$G [T_k^i(\mathbf{8}) \rightarrow M_k + \bar{M}^i] = G_0 \text{Tr}[\tilde{T}\mathcal{M}] + G_D \text{Tr}[\tilde{T}(\lambda_8\mathcal{M} + \mathcal{M}\lambda_8)] + G_F \text{Tr}[\tilde{T}(\lambda_8\mathcal{M} - \mathcal{M}\lambda_8)]. \quad (7)$$

Explicitly, for the nine tetraquark decays

$$G(T_1^1 \rightarrow M_1 + \bar{M}^1) = G_0 + 2G_D, \quad (8)$$

$$G(T_2^2 \rightarrow M_2 + \bar{M}^2) = G_0 + 2G_D, \quad (9)$$

$$G(T_3^3 \rightarrow M_3 + \bar{M}^3) = G_0 - 4G_D, \quad (10)$$

$$G(T_1^2 \rightarrow M_1 + \bar{M}^2) = G_0 + 2G_D, \quad (11)$$

$$G(T_2^1 \rightarrow M_2 + \bar{M}^1) = G_0 + 2G_D, \quad (12)$$

$$G(T_1^3 \rightarrow M_1 + \bar{M}^3) = G_0 - G_D + 3G_F, \quad (13)$$

$$G(T_2^3 \rightarrow M_2 + \bar{M}^3) = G_0 - G_D + 3G_F, \quad (14)$$

$$G(T_3^1 \rightarrow M_3 + \bar{M}^1) = G_0 - G_D - 3G_F, \quad (15)$$

$$G(T_3^2 \rightarrow M_3 + \bar{M}^2) = G_0 - G_D - 3G_F. \quad (16)$$

Nine decays and three coupling constants. So, six sum rules or relations,

$$G(T_1^1 \rightarrow M_1 + \bar{M}^1) = G(T_2^2 \rightarrow M_2 + \bar{M}^2) = G(T_1^2 \rightarrow M_1 + \bar{M}^2) = G(T_2^1 \rightarrow M_2 + \bar{M}^1); \quad (17)$$

$$G(T_1^3 \rightarrow M_1 + \bar{M}^3) = G(T_2^3 \rightarrow M_2 + \bar{M}^3), \quad G(T_3^1 \rightarrow M_3 + \bar{M}^1) = G(T_3^2 \rightarrow M_3 + \bar{M}^2); \quad (18)$$

$$G(T_2^2 \rightarrow M_2 + \bar{M}^2) + G(T_3^3 \rightarrow M_3 + \bar{M}^3) = G(T_1^3 \rightarrow M_1 + \bar{M}^3) + G(T_3^1 \rightarrow M_3 + \bar{M}^1). \quad (19)$$

If and when such tetraquarks are observed one can extract the coupling constants from the observed decay rates. For a  $J^P = 0^+$  tetraquark decaying into two pseudoscalar mesons the decay width is

$$\Gamma = \frac{1}{8\pi} \frac{k}{M^2} |A|^2, \quad (20)$$

where  $M$  is the tetraquark mass,  $k$  is the momentum of a decay particle and  $A$  is the transition amplitude. For  $s$ -wave decay ( $0^+ \rightarrow 0^- + 0^-$ )  $A = GM$  with  $G$  the coupling constant.

#### IV. CONCLUDING REMARKS

To date the tetraquarks  $T[(\bar{Q}q)(Q\bar{q})]$  have not been observed experimentally. However, their decays products, the heavy mesons (for example  $D_S^+ = c\bar{s}$ ,  $B^+ = u\bar{b}$ , etc.) have been observed [9]. It is conceivable that this type of tetraquark may be seen as a resonance in the heavy quark particle-antiparticle mass spectra, for example,  $D_S^+ + D_S^-$ ,  $D^0 + \bar{D}^0$ , etc. Another possibility is that they may be observed through their decay into light meson plus anti meson pairs, for example,  $q_k \bar{q}^l + q_l \bar{q}^i$  (see Fig. 1). These are possible through annihilation of  $\bar{Q}Q$  through gluons into  $\bar{q}q$ . However, such decays are likely to be OZI [6–8] suppressed.

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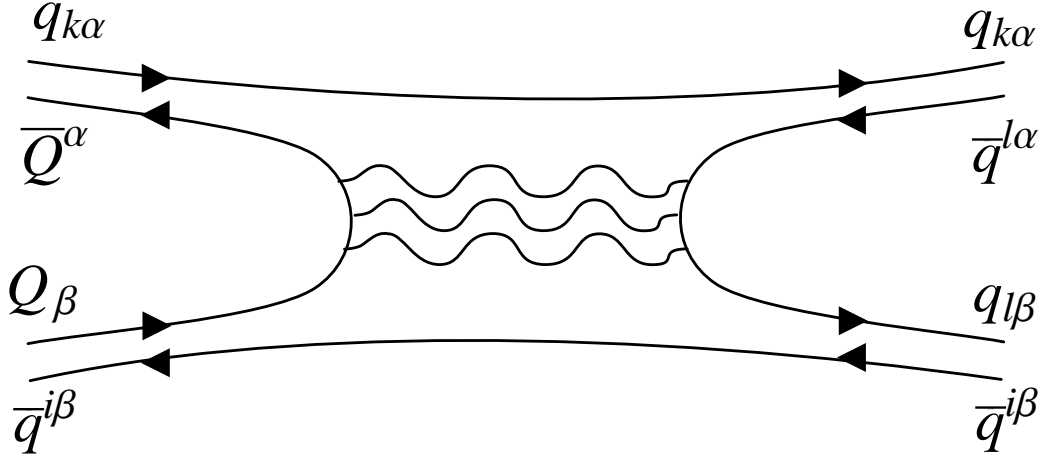


FIG. 1: Tetraquarks  $T[(\bar{Q}q)(Q\bar{q})]$  observed through their OZI suppressed decay into light meson plus anti meson pairs,  $q_k\bar{q}^l + q_l\bar{q}^i$ . Greek letters represent the color indices and latin letters represent flavor indices.